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Constrained cracking in cross-ply laminates

Y. KORCZYNSKYJ, J. G. MORLEY
Department of Metallurgy and Materials Science and Wolfson Institute of Interfacial Technology, University of Nottingham, Nottingham, UK

An analysis of the mechanics of crack growth, parallel to the fibre alignment, in a transverse-ply in a 0°/90° cross-plied laminate has been developed. This is based on a simple description of the form of the strain field around a crack of arbitrary length in a transverse-ply. The analysis predicts transverse-ply cracking strains which correspond closely with observed values for a range of laminate characteristics. Various techniques for crack suppression are suggested by the analysis which also links the properties of the laminate with other possible modes of failure.

1. Introduction

When a fibre reinforced polymeric matrix sheet is called upon to support transverse tensile loads in addition to longitudinal ones, it is usual to meet this requirement by arranging for the structure to be built up from a number of layers or plies. The fibres are unidirectionally aligned within each ply but the plies are oriented in different directions. This type of arrangement provides a convenient means whereby the degree of anisotropy in the mechanical properties of the laminate can be controlled.

It is observed that, when laminates are placed under a tensile load, fibre debonding and matrix cracking occurs preferentially in the plies oriented transversely to the loading direction. Damage of this type occurs at stress levels very much less than the ultimate tensile strength of the laminate. The cracks may render the structure unsuitable for liquid containment and may enhance further degradation by facilitating the penetration of water and other deleterious substances. The simplest form of cross-plied laminate is one in which the plies are oriented at 90° to each other. When tensile loads are applied in the direction of the longitudinal plies cracking is observed to occur in the transverse plies. Data on the incidence of cracking in the transverse plies of carbon fibre and glass fibre epoxy resin systems of this type have recently been published by Bailey, Curtis and Parvizi [1].

These authors developed an analysis of the cracking of the transverse plies which is an extension of the theory of multiple matrix cracking in unidirectional fibrous composites previously put forward by Aveston, Cooper and Kelly [2] and Aveston and Kelly [3]. This is based on a consideration of the potential energy difference between the initial uncracked condition and one in which a transverse crack has propagated across the entire section of the material. Specifically it is assumed that, per unit area of composite,

$$2\gamma_m V_m + \gamma_{db} + U_s + \Delta U_f \leqslant \Delta W + \Delta U_m \qquad (1)$$

where ΔW is the work done by the applied stress, ΔU_m is the loss of elastic strain energy by the matrix, γ_m is the surface energy of the matrix and V_m its volume fraction, γ_{db} is the fibre debonding energy, U_s is the frictional losses occurring at the fibre–matrix interface and ΔU_f is the increase in strain energy of the crack bridging fibres. The argument has been applied to laminates using a modified shear lag analysis [4] and to cracking constraint effects in 0°/90°0° laminates [5]. Bailey, Curtis and Parvizi [1] have considered the effect of thermal stresses developed during composite fabrication and show that the transverse-ply cracking strain, deduced from simple constraint theory, is reduced by an amount equal to the thermally developed tensile strain in the transverse-ply. These authors have also considered the effects of strains produced by differences in Poisson's ratio between the plies on the incidence of cracking parallel to the fibres. They showed that this could account for the observed longitudinal splitting of the 0° plies in cross-plied glass fibre reinforced laminates.

2. The mechanics of crack extension in transverse-plies

In this paper an alternative theoretical argument relating to the suppression of crack growth parallel to the fibres in the transverse-plies of 0°/90° cross-plied lamainates is considered. It is assumed that pre-existing cracks are present in the transverse-plies. These cracks are oriented parallel to the fibres and have a finite length in that direction. They are assumed to extend through the full thickness of the transverse-ply since the analytical treatment is two-dimensional. The cracks propagate in a direction parallel to the fibres at a critical stress level when a tensile load is applied in the direction of the longitudinal-plies. The mechanics of crack propagation in the transverse-plies is influenced by the presence of the still intact crack bridging longitudinal-plies and the characteristics of the interface between adjacent plies.

Firstly, an estimate is made of the effective size of the initial cracks in the transverse-plies from a knowledge of the work of fracture G_{ct} of an individual ply when loaded transversely in tension so that crack growth parallel to the fibres occurs. This estimate is based on the Griffith analysis assuming that only the transverse elastic modulus of the ply need be considered in the calculation, i.e.

$$\frac{\pi a \sigma_t^2}{E_t} = G_{ct} \tag{2}$$

where $2a$ is the crack length, σ_t the transverse stress, E_t the transverse elastic modulus and G_{ct} the the strain energy release rate for crack growth parallel to the fibres.

The mechanism by which unstable crack extension is inhibited is derived from a physical model previously developed to deal with transverse matrix crack growth in unidirectional fibrous composites, [6, 7], loaded in the direction of fibre alignment. The form of strain field which would develop in the transverse-ply in the absence of any interaction between it and the longitudinal-plies is shown in Fig. 1. The strain is assumed to increase linearly from the face of the crack to the edge of an elliptical zone drawn round the crack. Strains are assumed to develop only in the loading direction so that the zone can be considered to behave

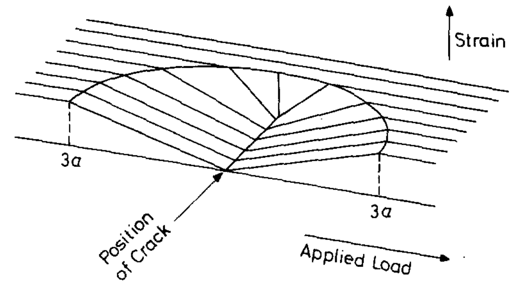

Figure 1 Idealized strain field around a crack aligned parallel with the fibres in a single transverse-ply. The crack extends throughout the thickness of the ply and has an arbitrary length, 2a, parallel with the fibres. The load is applied in the plane of the ply and perpendicular to the orientation of the fibres. The strains developed in the material in the direction of the applied load are indicated on the vertical axis. For clarity only one-half of the strain field is shown. The plane of the crack is perpendicular to the face of the ply.

as a parallel array of independent segments. The strain energy released by the material within the zone is readily computed on this basis and the size of the zone is chosen so the strain energy released by the elastic relaxation of the material within it, is the same as that calculated by Griffith for a crack of the same length in an isotropic elastic material at the same bulk strain value. On this basis the major axis of the elliptical zone is three times the crack length.

The analysis previously developed [6, 7], considers a matrix crack to be bridged by cylindrical fibres and the strain distribution within a parallel sided segment of the material aligned in the loading direction is illustrated in Fig. 2. For this condition the fibres carry their maximum strain (and stress) where they cross the matrix crack. The distance along a fibre measured from the position of the crack is given by x. The slope of the line VQ is thus given by,

$$\frac{d\epsilon_f}{dx} = \frac{2\tau}{E_f r} \tag{3}$$

where ϵ_f is the strain carried by the fibres and τ is the interfacial shear stress. The slope of OQ is given by,

$$\frac{d\epsilon_m}{dx} = \frac{2V_f\tau}{E_m V_m r} + \frac{\epsilon_\beta}{L_3} \tag{4}$$

where ϵ_m is the strain carried by the matrix and $2r$ is the fibre diameter. E_f, E_m, V_f and V_m have their usual meanings and the general strain carried by the composite outside the elliptical zone around the crack is ϵ_β. L_3 is the length of a segment in one quadrant of the elliptical zone and is given by,

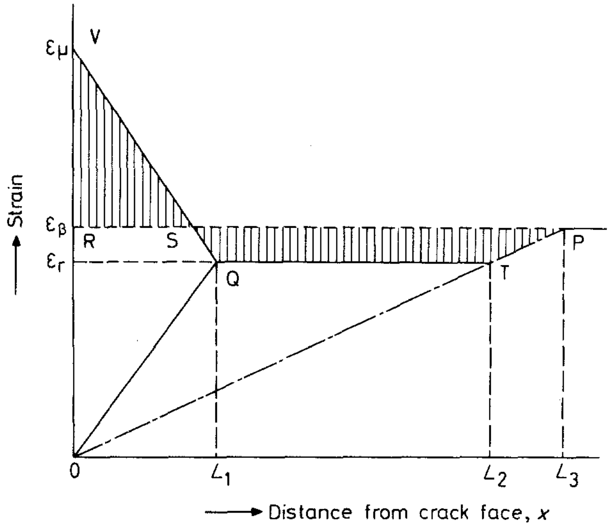

Figure 2 Assumed strain distribution within a parallel side segment perpendicular to a transverse matrix crack in a unidirectionally reinforced fibrous composite.

$$L_3 = 3(a^2 - y^2)^{1/2} \tag{5}$$

where y is the distance from the centre of the crack to the section considered. Since the redistributions of strain are assumed to be confined to the region within the elliptical zone, the total length of a crack bridging fibre traversing the zone must be unaffected by the presence of the crack. It follows that the areas shown shaded in Fig. 2 are equal and this boundary condition together with Equations 3 and 4 enable equations defining the strain distribution along any section within the elliptical zone perpendicular to the plane of the crack to be derived. These are,

$$\begin{aligned} \epsilon_r &= \{\epsilon_\beta L_3/[Q(P+\epsilon_\beta/L_3)^{-2} + L_3/\epsilon_\beta]\}^{1/2} \\ \epsilon_\mu &= L_1(P + Q + \epsilon_\beta/L_3) \\ L_1 &= \epsilon_r(P+\epsilon_\beta/L_3)^{-1} \quad L_2 = L_3\epsilon_r/\epsilon_\beta \end{aligned} \tag{6}$$

where $P = 2V_f\tau/(E_m V_m)r$ and $Q = 2\tau/E_f r$. ϵ_r, ϵ_μ, ϵ_β, L_1, L_2 and L_3 are defined in Fig. 2.

Following this argument, the strain field around the crack in the transverse-ply is now assumed to be modified by interactions between adjacent longitudinal- and transverse-plies. The interaction is considered to take place through the interlaminar interface at a uniform value of shear stress transfer, (although other stress transfer functions could be used). It follows that the unfractured longitudinal-ply carries its maximum stress (and corresponding strain) at the point where it bridges the crack in the transverse-ply. Again strains are assumed to occur only in the direction of the applied tensile load, i.e. (parallel with the fibres in the longitudinal-ply). Deformation due to the presence of the crack in the transverse-ply is still considered to be confined to the elliptical zone around the crack as defined above. Within this zone the material is thus considered to behave as a number of parallel independent segments. The form of the strain field developed on the basis of these arguments is illustrated in Fig. 3. The equations defining the strain field are derived as follows.

2.1. Two-ply system

The simplest arrangement to consider is that of a two-ply system as illustrated in Fig. 4. First consider the strain distributions within the plies in the vicinity of the crack in the transverse-ply. The argument is first confined to the behaviour of a single element of width dy (measured in a direction parallel to the crack axis).

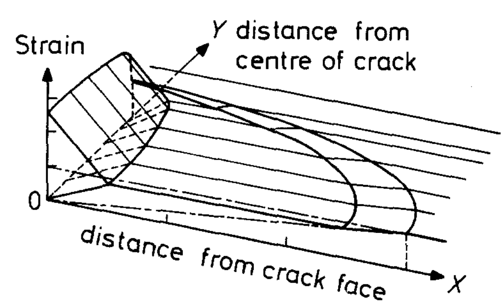

Figure 3 Quadrant of strain field around a crack parallel to the fibres in a transverse-ply. The centre of the crack is at "O". The O–Y axis indicates the position of the crack, its length and its direction of propagation. The crack is assumed to extend through the full thickness of the transverse-ply. The local increase in strain carried by the intact longitudinal-ply where it bridges the crack is indicated.

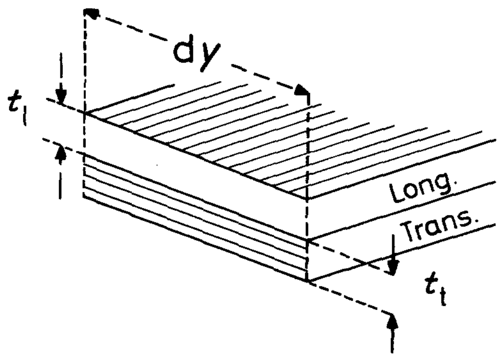

Figure 4 Stress transfer between transverse- and longitudinal-plies in two-ply 0°/90° system.

If t_l and t_t are the actual longitudinal- and transverse-ply thicknesses, respectively, then their effective volume fractions (T_l and T_t) are given by,

$$T_l = \frac{t_l}{t_l + t_t} \quad \text{and} \quad T_t = \frac{t_t}{t_l + t_t}. \quad (7)$$

The load lost from the longitudinal-ply by shear stress transfer over the interply interface to the transverse-ply over a distance x^1 must equal the gain in the load carried by the transverse-ply. Defining the change in stress in the longitudinal-ply over the distance x^1 as $(\sigma_l - \sigma_{lx^1})$ and the corresponding change in the stress carried by the transverse-ply as σ_{tx^1}. The stress in the transverse-ply is of course zero at the crack face.

Hence, $(\sigma_l - \sigma_{lx^1})t_l\mathrm{d}y = \sigma_{tx^1}t_t\mathrm{d}y$ and the strain gradient in the longitudinal-ply is given by,

$$\frac{\mathrm{d}\epsilon_l}{\mathrm{d}x} = \frac{(\sigma_l - \sigma_{lx^1})}{E_l x^1} \quad (8)$$

so that,

$$\frac{\mathrm{d}\epsilon_l}{\mathrm{d}x} = -\frac{\epsilon_{tx^1}E_t T_t}{E_l T_l x^1} = Q \quad (9)$$

where E_l is the elastic modulus of the longitudinal-ply measured in the direction of loading (parallel with the fibres) and E_t is the elastic modulus of the transverse-ply measured in the direction of loading (transversely to the fibre alignment). ϵ_{tx^1} is the strain in the transverse-ply corresponding to the stress σ_{tx^1}. Equating the shear stress τ acting at the interply interface with the increase in load carried by the transverse-ply over the distance x^1,

$$\tau x^1 \mathrm{d}y = \sigma_{tx^1}t_t\mathrm{d}y = \sigma_{tx^1}T_t(t_l + t_t)\mathrm{d}y \quad (10)$$

so that,

$$x^1 = \frac{\epsilon_{tx^1}E_t t_t}{\tau} = \frac{\epsilon_{tx^1}E_t T_t(t_l + t_t)}{\tau} \quad (11)$$

and,

$$\frac{\mathrm{d}\epsilon_t}{\mathrm{d}x} = \frac{\epsilon_{tx^1}}{x^1} = \frac{\tau}{E_t t_t} = \frac{\tau}{E_t T_t(t_l + t_t)} = P. \quad (12)$$

From Equations (9) and (11),

$$\frac{\mathrm{d}\epsilon_l}{\mathrm{d}x} = -\frac{\tau}{T_l E_l(t_l + t_t)} = Q. \quad (13)$$

2.2. Three cross-plied system

For a three cross-plied system two geometrical arrangements are possible as indicated in Fig. 5. Again t_t and t_l are the transverse- and longitudinal-ply thicknesses. Interaction now takes place across the two interply interfaces.

For case (a),

$$T_t(\mathrm{a}) = \frac{2t_t}{(2t_t + t_l)} \quad (14\mathrm{a})$$

and for (b),

$$T_t(\mathrm{b}) = \frac{t_t}{(2t_l + t_t)}. \quad (14\mathrm{b})$$

T_l is similarly defined for both conditions.

Again equating the total load applied at the position of the crack with that applied a distance x^1 away,

$$\Delta\sigma_l T_l = -\sigma_{tx^1}T_t \quad (15)$$

so that,

$$\frac{\mathrm{d}\epsilon_l}{\mathrm{d}x} = \frac{\Delta\epsilon_l}{x'} = -\frac{\epsilon_{tx^1}E_t T_t}{E_l T_l x^1}. \quad (16)$$

Now x^1 is obtained in terms of the physical parameters of the system by reference to Fig. 5.

For the first case (Fig. 5a), dividing the composite into two equal "unit cells" along A–A′ gives for the total force transferred across the interply interface.

$$\mathrm{d}y x^1(\mathrm{a})\tau = \sigma_{tx^1}t_t\mathrm{d}y. \quad (17)$$

Thus,

$$x^1(\mathrm{a}) = \epsilon_{tx^1}E_t t_t/\tau$$

which from the definition of T_t(a) in Equation 14a gives,

$$x^1(\mathrm{a}) = \frac{\epsilon_{tx^1}E_t T_t(\mathrm{a})(2t_t + t_l)}{2\tau}. \quad (18)$$

Hence,

$$\frac{\mathrm{d}\epsilon_t(\mathrm{a})}{\mathrm{d}x} = \frac{\epsilon_{tx^1}}{x^1} = \frac{2\tau}{E_t T_t(\mathrm{a})(2t_t + t_l)} \quad (19)$$

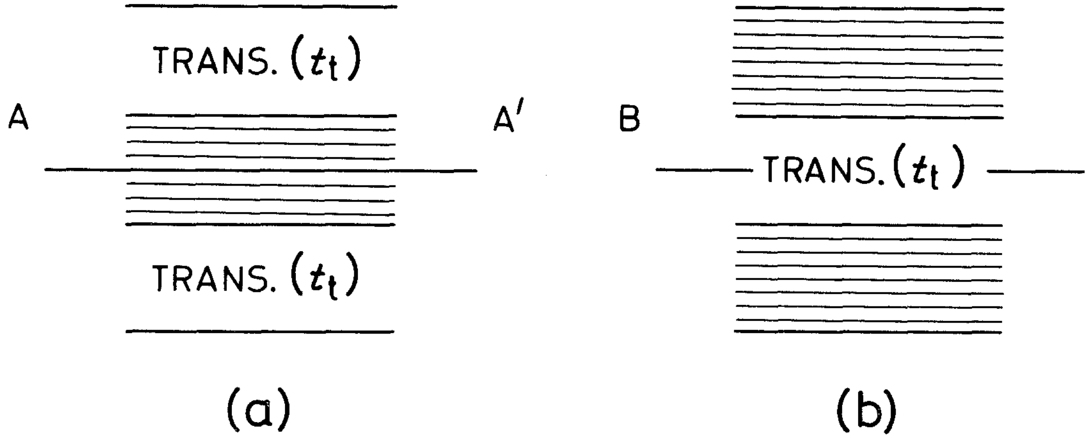

Figure 5 3-ply system.

and,

$$\frac{d\epsilon_l(a)}{dx} = \frac{-2\tau}{E_l T_l(a)(2t_t + t_l)}. \tag{20}$$

In the second case (Fig. 5b),

$$dy x^1(b)\tau = \sigma_{tx^1}(t_t/2)dy \tag{21}$$

so that,

$$x^1(b) = \frac{\epsilon_{tx^1} E_t t_t}{2\tau}, \tag{22}$$

which from Equation 14b gives,

$$x^1(b) = \frac{\epsilon_{tx^1} E_t T_t(b)(2t_l + t_t)}{2\tau} \tag{23}$$

thus,

$$\frac{d\epsilon_t(b)}{dx} = \frac{2\tau}{E_t T_t(b)(2t_l + t_t)} \tag{24}$$

and

$$\frac{d\epsilon_l(b)}{dx} = \frac{-2\tau}{E_l T_l(b)(2t_l + t_t)}. \tag{25}$$

Equations 23 to 25 are of identical form to Equations 18 to 20, so that the notations (a) and (b) are unnecessary even though $T_t(a)$ may differ from $T_t(b)$.

Note that for the same total laminate thickness in Fig. 5a and b and for $t_l = t_t$: $T_t(a) = 2/3$ and $T_t(b) = 1/3$. Hence, for this condition from Equations 19 and 24.

$$\frac{d\epsilon_t(a)}{dx} = \frac{1}{2} = \frac{d\epsilon_t(b)}{dx} \tag{26}$$

since,

$$T_t(a) = 2T_t(b) \tag{27}$$

and from Equations 20 and 25,

$$\frac{d\epsilon_l(a)}{dx} = \frac{2d\epsilon_l(b)}{dx} \tag{28}$$

since

$$T_l(a) = \tfrac{1}{2}T_l(b). \tag{29}$$

2.3. Multiply laminates

Consider N longitudinal-plies each of thickness t_l and M transverse-plies each of thickness t_t so that $M = N \pm 1$ or $M = N$. If M and N are large the effects due to surface plies can be neglected and the laminate may be considered as being divided up "into unit cells" each consisting of two half-ply thicknesses. The analysis which follows is assumed to be representative of a situation in which the initial cracks are of the same length and are considered to be present in the same plane in all of the transverse-plies.

Equating as before the total loads at the position of the crack and at an arbitrary distance x^1 away from the crack,

$$-\frac{d\epsilon_l}{dx} = \frac{\epsilon_{tx^1} E_t T_t}{E_l T_l x^1} \tag{30}$$

where

$$T_t = \frac{Nt_l}{(Nt_l + Mt_t)}. \tag{31}$$

and

$$T_l = \frac{Nt_l}{(Nt_l + Mt_t)}. \tag{32}$$

The total force transferred across the interface in the unit cell is

$$dy x^1 \tau = \frac{\sigma_{tx^1}\, dy t_t}{2} \tag{33}$$

so that

$$x^1 = \frac{\epsilon_{tx^1} E_t T_t (Nt_l + Mt_t)}{2\tau M}. \qquad (34)$$

Note that for $N = 2$ and $M = 1$, case (b) of the three-ply laminate, Equation 34 reduces to Equation 23 but for $M = 2$ and $N = 1$, case (a) of the three-ply laminate, it contains an additional factor of $\frac{1}{2}$ compared with Equation 18. This is due to the neglect of the effect of the surface plies inherent in the "unit cell" approach to the problem. The strain gradients in the multiply laminate are given by,

$$\frac{d\epsilon_t}{dx} = \frac{2\tau M}{E_t T_t (Nt_l + Mt_t)} \qquad (35)$$

and

$$\frac{d\epsilon_l}{dx} = \frac{-2\tau M}{E_l T_l (Nt_l + Mt_t)} \qquad (36)$$

Equations 35 and 36 can be expressed in terms of total laminate thickness T_{tot} where,

$$T_{tot} = Nt_l + Mt_t \qquad (37)$$

so that from Equation 34

$$x^1 = \frac{\epsilon_{tx^1} E_t T_t T_{tot}}{K\tau}$$

where $K = 2M$.
Hence,

$$\frac{d\epsilon_t}{dx} = \frac{k\tau}{E_t T_t T_{tot}} \qquad (38)$$

and

$$\frac{d\epsilon_l}{dx} = \frac{-K\tau}{E_l T_l T_{tot}}. \qquad (39)$$

Consideration of the previous analysis shows that,

$K = 1$ for a two-ply laminate.

$K = 2$ for a three-ply laminate.

$K = 2M$ for a laminate consisting of a large number of individual plies.

Note that although the rate of stress transfer appears to increase with an increase in M, the number of transverse-plies, the assumptions made in the "unit cell" approach imply a corresponding increase in the number of longitudinal-plies.

Comparing equations 38 and 39 for the laminate with the values of P and Q given in Equations 3 and 4 for a transverse crack in a fibre reinforced matrix,

$$P\,(\text{fibre system}) = \frac{2V_f\tau}{E_m V_m r} \qquad (40)$$

$$\text{P (laminate system)} = \frac{K\tau}{E_t T_t T_{tot}} \qquad (41)$$

and,

$$Q\,(\text{fibre system}) = \frac{2\tau}{E_f r} \qquad (42)$$

$$\text{Q (laminate system)} = \frac{K\tau}{E_l T_l T_{tot}}. \qquad (43)$$

Thus in order to transpose from a fibre reinforced system to a laminate system the following substitutions are made: T_l for V_f, T_t for V_m, E_l for E_f, E_t for E_m and $K/T_l T_{tot}$ for $2/r$.

The rate of release of strain energy with increasing crack length has been calculated for a fibre reinforced system [7]. The energy released by a parallel sided section of the composite having a width δy, with unit thickness and positioned an arbitrary distance y from the centre of the crack depends on the geometrical form of the strain distribution within it as illustrated in Fig. 2. Its numerical value is given by,

$$\begin{aligned}\delta W_{Ry} = \{&\tfrac{1}{2}E_c\epsilon_\beta^2 L_3 - \tfrac{1}{6}E_c\epsilon_\beta^2(L_3^3 - L_2^3)/L_3^2 \\ &- \tfrac{1}{2}E_c\epsilon_r^2(L_2 - L_1) - \tfrac{1}{6}E_m\epsilon_r^2 L_1 \\ &- \tfrac{1}{6}V_f E_f(\epsilon_\mu^2 + \epsilon_\mu\epsilon_r + \epsilon_r^2)L_1\}\delta y\end{aligned} \qquad (44)$$

where $E_c = E_f V_f + E_m V_m$.

In place of E_c, the elastic modulus of the fibre composite, $(E_l T_l + E_t T_t)$ the longitudinal elastic modulus of the laminate can be substituted. Note that the fibre diameter $2r$ does not enter Equation 44 directly. The variables ϵ_μ etc. depend on the fibre diameter through the stress transfer rates P and Q. Thus δW_{Ry} can be obtained for the laminate by making the appropriate modifications to P and Q in Equation 6. By numerically integrating Equation 44 the strain energy released by the laminate over the entire elliptical zone around a crack of arbitrary length is obtained. The rate of release of strain energy with increasing crack length is then obtained by numerically differentiating the strain energy released at successive crack lengths.

For a fibre reinforced system energy is absorbed by displacement occurring at the fibre–matrix interface due to elastic strain differences. If the interface has a limiting shear strength τ the amount of energy absorbed in one sector of the elliptical zone is given by,

$$\delta W_{Ay} = \frac{V_f\tau\epsilon_\mu L_1^2\delta y}{3r}. \qquad (45)$$

Hence, by transposing, the energy absorbed by similar displacements at the interlaminar interface is given by,

$$\delta W_{Ay} = \frac{K\tau\epsilon_{\mu}L_1^2\delta y}{6T_{tot}}. \quad (46)$$

The rate of absorption of energy with increasing crack length can then be obtained using the same technique as used to obtain the values of strain energy release rates.

It is assumed that a crack of arbitrary length in a transverse-ply will become unstable when the rate of release of strain energy with increasing crack length equals the amount of energy absorbed during crack extension. The energy release rate can be obtained from Equation 44. Energy is absorbed by rupturing the transverse-ply and this is expressed as G_{ct} (Equation 2). The energy absorbed at the interlaminar interface during crack extension can be obtained from Equation 46. Hence the composite strain at which a crack of arbitrary length in the transverse-ply becomes unstable can be calculated.

3. Application of the threoretical model to the observed behaviour of three-ply laminates

Bailey, Curtis and Parvizi [1] have studied the cracking behaviour of 0°/90°/0° and 90°/0°/90° three-ply laminates having various ply thicknesses. The onset of cracking in the transverse plies was obtained from acoustic emission data as a monotonically increasing longitudinal load was applied. Observations were made on both carbon fibre reinforced epoxy resin (CFRP) and glass fibre reinforced epoxy resin (GFRP).

A single ply of CFRP loaded transversely was observed to fail at a strain, ϵ_{tu}, of 0.0048 ± 0.0005 with an effective surface energy of 77 ± 20 J m^{-2}. From this data and that of the transverse elastic modulus (8.3 ± 0.3 GPa) and assuming the validity of Equation 2, cracks having lengths ranging from about 0.2 to 0.8 mm can be assumed to be present in the transverse-plies. Similar failing strain values, ϵ_{tu}, are observed for unidirectional GFRP laminates loaded in tension perpendicular to the fibre alignment. Cracks are also assumed to be present in these materials (see Section 3.2).

According to the theoretical argument set out in Section 2, the cracks in the transverse-plies would be expected to propagate at enhanced strain values because of the presence of the longitudinal plies. The strain values for unstable propagation of the cracks has been calculated for the various experimental samples tested by Bailey *et al.* [1] using the theory set out in Section 2 and assuming various initial crack lengths and corresponding failing strains. These calculated values were then compared with the experimental observations previously obtained.

3.1. CFRP cross-ply laminates

Bailey *et al.* [1] obtained experimental data on transverse-ply cracking in 0°/90°/0° carbon fibre symmetrical laminates with constant outer ply thicknesses. In all of these CFRP laminates the outer plies had a constant thickness of $\frac{1}{2}$mm with an inner ply thickness of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$ or 1 mm. Because the thermal expansion coefficients of the plies are very different in the longitudinal direction compared with the transverse direction, residual thermal stresses are developed after curing. Bailey *et al.* [1] determined the magnitude of these stresses experimentally. These values are added to the strain developed in the sample when cracking is initiated to give the true strain developed in the transverse-ply for this condition. The shear strength of the interply interface is assumed by the present authors to be 10 MN m^{-1} since this seems representative of previous direct measurements. This value was taken to define τ, the interply shear stress stransfer rate used in the theoretical analysis. Good agreement between the cracking strain values predicted by the analysis and the experimental values is obtained (Fig. 6) if the cracking strain of the transverse-ply (tested independently) is assumed to be 0.00518. This is within the range of experimental observations (0.0048 ± 0.0005) and corresponds to a half crack length of 0.22 mm, assuming the validity of Equation 2. The calculated cracking strain values are shown in Fig. 6 and should approach 0.005 18 as the total thickness of the laminate becomes very large so that the effect of the surface plies becomes negligible.

Bailey *et al.* [1] also studied the onset of crack-

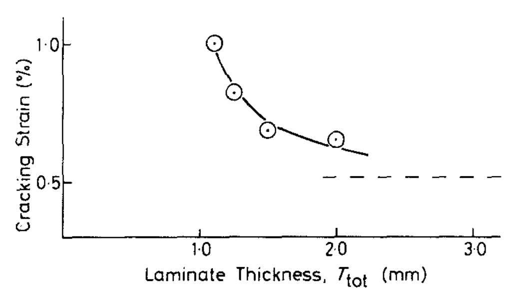

Figure 6 0°/90°/0° CFRP laminates. Experimental values of transverse-ply cracking strain compared with predicted values, ϵ_{tu} = 0.005 18.

ing in the outer transverse-plies of 90°/0°/90° CFRP laminates. The thickness of each of the two outer plies was held constant at 0.5 mm and the thickness of the inner (0°) longitudinal-ply was varied as before. Again the residual thermal strains developed in the transverse-plies in the direction of the applied load (longitudinal direction) were calculated.

The value of the interply shear stress transfer rate, τ, and the intrinsic failing strain of the transverse-plies, $\epsilon_{tu} = 0.00518$, were assumed to be the same as for the 0°/90°/0° condition. The enhancement of the transverse-ply cracking strain by the central longitudinal-ply in the 90°/0°/90° laminate was then calculated as before using the theory set out in Section 2 of this paper. The theoretical laminate strains for transverse-ply crack extension calculated in this way are compared in Fig. 7 with the sum of the initial thermal strains, and the observed additional mechanical strains to which the specimens were subjected when transverse cracking was observed. The basic cracking strain of the transverse laminate (ϵ_{tu}) would be expected at a total laminate thickness of 1.0 mm since at this laminate thickness the longitudinal-ply thickness becomes zero. According to the present theory the laminate strain for simultaneous crack extension in both of the outer transverse-plies is predicted to increase with increasing laminate thickness since this corresponds to a progressive increase in the thickness of the central longitudinal-ply.

It is interesting to note that the theory presented here predicts the same cracking strain for the transverse-plies in both the 0°/90°/0° and 90°/0°/90° composites when the central ply has twice the thickness of the outer plies. This occurs because for these conditions there are equal volume fractions of fibres aligned in the two directions and

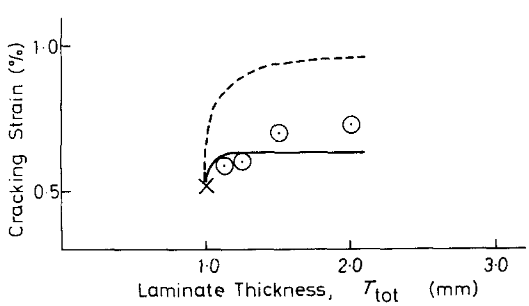

Figure 7 90°/0°/90° CFRP laminate. Upper and lower limits of computed transverse-ply cracking strains (fracture of one or both transverse-plies) compared with experimental values.

the same number of interfaces, (two) for stress transfer and energy absorption. Both systems can therefore be divided into identical unit cells. These conditions are met for laminates having a total thickness of 2 mm, but not for other laminate thicknesses. Hence, the two continuous curves shown in Figs 6 and 7 computed from this theory intersect at the point corresponding to a total laminate thickness of 2 mm.

In the development of the theory for the cracking behaviour of 90°/0°/90° laminates it is assumed that cracks are present in both outer transverse-plies and that these are of the same size and in the in the same plane and propagate simultaneously at the same laminate strain value. This is unlikely to be the case unless very large number of flaws are present. Otherwise cracking will be initiated at the most severe flaw in one or other of the transverse outer plies. This may take place at a higher laminate strain than that predicted by the theory for the following reason. At the onset of the unstable crack extension in one or other outer transverse-ply, the whole of the central 0° ply will be effective in suppressing its extension. In addition a small contribution to crack suppression will be made by the still intact transverse-ply. If it is first assumed that no flaws are present in the still intact outer transverse-ply, the laminate strain at which the crack in the damaged transverse-ply will propagate can be calculated using the analysis given in Section 2. if this is done laminate strains approaching 0.01 are obtained. (Fig. 7, dashed curve). However, this represents an upper limiting condition which will not be approached because of the presence of flaws which must be present in the still intact transverse-ply. Hence, experimentally, cracking would be expected to be observed at strains between these upper and lower limits the precise value being set by the form of the flaw distribution in the transverse-plies. The available experimental data conforms with this supposition.

3.2. Glass fibre laminates

The theoretical analysis set out in Section 2 has also been used to predict the cracking behaviour of 90°/0°/90° cross-plied glass fibre reinforced epoxy resin laminates studied by Bailey *et al.* [1] who prepared and tested two groups of experimental samples. One group had two outer transverse plies, each having a constant thickness of 0.5 mm and an inner ply which varied in thickness between 0.3 mm and 2.65 mm. The second group had a constant inner ply thickness of 1 mm with symmetrical transverse outer plies on each side of the laminate in thicknesses between 0.15 and 1.5 mm. These specimens were loaded in tension, with the fibres in the outer plies arranged transversely to the direction of loading and the laminate strain at which cracking was observed in the transverse-plies was noted.

Bailey *et al.* [1] give the fracture strain of a unidirectional GFRP laminate loaded transversely as 0.005. From this figure and from the elastic modulus and fracture surface energy values given, the half-length of the intrinsic cracks assumed to be present and aligned in a direction parallel with the fibres in the outer transverse-plies of these laminates is calculated from Equation 2 to be 0.22 mm. The extension of these cracks will be suppressed by the inner longitudinal-ply as discussed for the CFRP laminates in Section 3.1. In Fig. 8 calculated values for the applied strain at which simultaneous unstable crack extension occurs in both outer plies according to the theory set out in Section 2 are given for the first group of GFRP laminates. The theoretical cracking strain values for the transverse-plies are plotted against the total thickness of the laminate. Two theoretical curves are drawn corresponding to 90° ply failing strains, ϵ_{tu}, of 0.005 and 0.004 (with corresponding initial flaw sizes of 0.22 and 0.34 mm). The interply interfacial shear stress transfer rate is again taken as $10\,\mathrm{MN\,m^{-2}}$.

The experimental data points corrected for thermal strains are shown in Fig. 8 together with the values predicted from the theory set out in Section 2. It is apparent from Fig. 8 that good agreement between the theory and the experimental values is obtained except for the case of laminates having thick outer transverse-plies with a thin inner longitudinal-ply. This lack of correlation with theoretical predictions for these samples was also noted by Bailey *et al.* [1] who use an alternative theoretical approach to describe transverse-ply cracking. The cracking strains of the transverse-plies in the second group of $90^\circ/0^\circ/90^\circ$ GFRP laminates examined by Bailey *et al.* [1] were observed to be rather higher than the basic fracture strain of single 90° ply test specimens (Fig. 9). These observed values are in agreement with those which would be expected from the theoretical analysis presented here.

As discussed above, the cracking strains of the outer transverse plies of the $90^\circ/0^\circ/90^\circ$ GFRP laminates illustrated in Figs 8 and 9 would be expected to vary depending on whether fracture occurs simultaneously in both plies, by sequential crack extension in both outer plies or as a consequence of crack extension occurring in only one ply. The cracking strains for the boundary conditions, (simultaneous cracking and single cracking) have been calculated as described above for CFRP laminates. The effect is negligible in the case of the GFRP laminates because of the much lower elastic modulus of the longitudinal GFRP plies.

Parvizi *et al.* [5] have also observed the strains for unstable crack extension in the transverse plies of $0^\circ/90^\circ/0^\circ$ GFRP laminates. In the case of these specimens the two outer 0° plies each had a constant thickness of 0.5 mm and the inner 90° ply thickness was varied between 0.1 and 4.0 mm. The

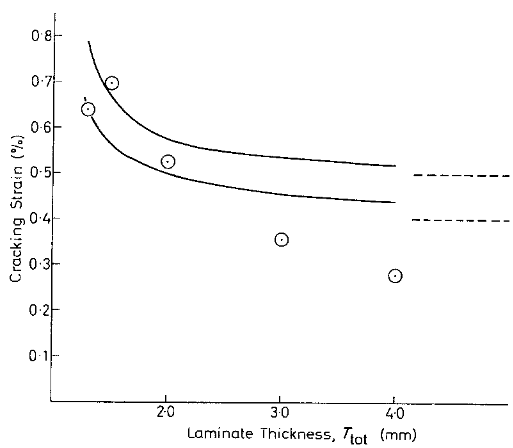

Figure 8 $90^\circ/0^\circ/90^\circ$ GFRP laminate. Upper and lower limits of computed transverse-ply cracking strains correspond to ϵ_{tu} values of 0.004 and 0.005. Inner ply thickness constant.

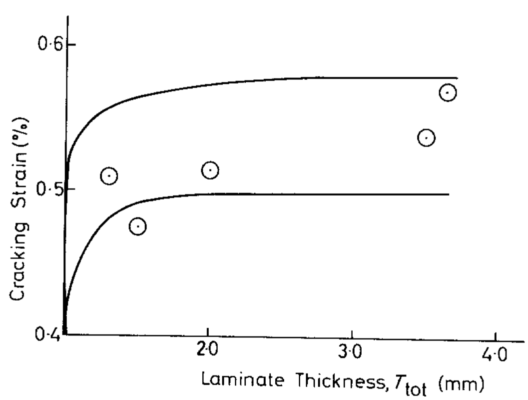

Figure 9 $90^\circ/0^\circ/90^\circ$ GFRP laminates. Upper and lower limits of computed transverse-ply cracking strains correspond to ϵ_{tu} values of 0.004 and 0.005. Thickness of outer plies constant.

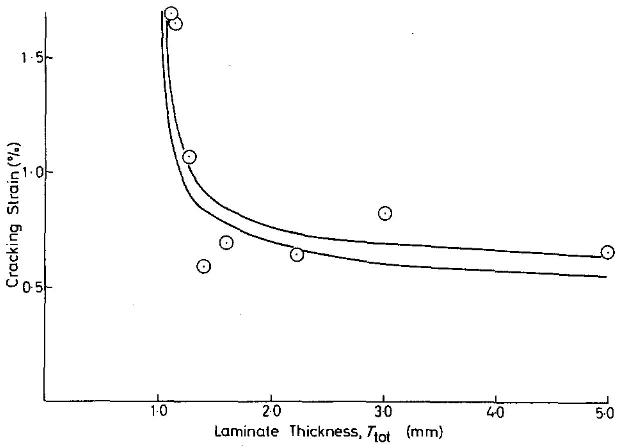

Figure 10 0°/90°/0° GFRP laminates. Upper and lower bounds of computed transverse-ply cracking strains correspond to $\tau = 20$ or $30\,\mathrm{MN\,m^{-2}}$ and $\epsilon_{tu} = 0.005$ or 0.006. Thickness of outer plies constant.

experimental values for the composite strain at which crack extension occurred in the central transverse-ply corrected for thermal stresses are shown in Fig. 10. Also shown in Fig. 10 are the limits, calculated from the theory presented here, for interply shear stress transfer rates of 20 and $30\,\mathrm{MN\,m^{-2}}$ and cracking strains for the transverse-ply alone, ϵ_{tu}, of 0.005 and 0.006. The theoretical cracking strains predicted from these values are in reasonable agreement with the experimental observations over the whole of the range of sample thicknesses investigated. The values of τ and ϵ_{tu} used in these calculations again correspond approximately to generally observed experimental values. A single experimental point strain value of 0.64% for initial cracking in a multiply CFRP laminate is given by Bailey *et al.* [1]. This compares closely with the value of 0.68% which was computed for the same system using the above theory with the effect of thermal stresses also taken into account.

4. Discussion

The theory presented here is based on:

(1) A first order estimate of the intrinsic flaw size parallel with the fibres in the transverse laminate and;

(2) the constraint on the growth of the flaw exerted by the longitudinal-ply or plies through a first order estimate of the form of the strain field around the crack and also of energy absorbing processes occurring at the interply interface.

It provides a continuous description of the influence of the longitudinal-0° ply (plies) on the cracking behaviour of the transverse-(90°) ply (plies) for all volume fractions of 0° plies. The failing strain of a single transverse-90° ply is thus the end point of the theoretical curves and corresponds to zero volume fraction of longitudinal plies. It is apparent from Figs 6 to 10, that the experimental initial cracking strain values for various cross-plied systems obtained by Bailey *et el.* [1] are in good agreement with the predictions of this theory when typical physical property values are used in the analysis. The only experimental observations which differ significantly from the theoretical predictions are the cracking strain values of laminates with thick outer 90° plies. For this system it was observed that the cracking strain of the outer plies fell to about half the value of the failing strain observed for a single transverse-ply tested independently. The present authors cannot put forward any physical explanation of these observations based on interactions between the longitudinal- and the transverse-plies.

Stress transfer between transverse- and longitudinal-plies is assumed to take place at a constant shear stress transfer value. Energy is absorbed as a consequence of displacements occurring at the interply interface. This is envisaged as occurring by frictional losses at debonded interfaces, and by the absorption of strain energy by shear displacements in the interply interface region. This latter process is analogous to the absorption of strain energy in the portion of the longitudinal-ply which bridges the crack thus reducing the amount of strain energy available to propagate the crack in the transverse-ply.

The analysis presented here is based on a consideration of a two-dimensional model and very simple assumptions are made about the form of the strain field and the mechanism of shear stress transfer and energy absorption at the interface. Nevertheless the analysis seems capable of predicting the experimental behaviour of cross-plied laminates and suggests processes by which premature cracking could be suppressed. One of these would be to reduce the thickness of the plies in the laminate, the effect of which was pointed out by Bailey *et al.* [1]. An alternative means of suppressing cracking would be to increase the value of the effective work of fracture of the transverse-plies, for crack propagation parallel to the fibres. This might be done, for example, by fabricating the transverse-plies from a number of thinner, slightly misaligned plies, so that crack propagation would involve more extensive fibre debonding and pull out.

Other types of failure mechanisms are predicted from the analysis. Fig. 2 indicates that high strain values will be developed in the longitudinal-plies where they bridge a crack in the transverse-plies. Thus, if the work of fracture of the transverse-plies is sufficiently large, failure of the longitudinal-plies may occur before unstable crack growth takes place in the transverse-plies. Also, if the longitudinal-plies are insufficient to support the load applied to the composite after failure of the transverse-plies, the laminate will fail by single fracture following the propagation of a transverse crack. Multiple cracking in the transverse-plies is, of course, observed when the longitudinal-plies are able to support all of the applied load. It is of interest to note that, when the laminate structure is such that cracking of the transverse-plies is severely inhibited, short (< 1 mm) non-propagating edge cracks can be observed in the transverse-plies [5].

The analysis put forward here deals with an idealized system in which the component parts of the composite structure have uniform properties. For these conditions unstable crack growth is predicted at a critical stress level. In the case of real composite materials there will be small-scale variations in material characteristics. Hence, under experimental conditions the predictions of the theory may be modified by small-scale variations in material properties of the order of the crack size considered. These issues have been discussed elsewhere [8] in the context of a unidirectionally reinforced brittle matrix composite material.

5. Conclusions

The mechanics of propagation of a crack of arbitrary length in a transverse-ply subjected to a transverse strain, in a cross-plied laminate subjected to tensile loading is considered. This analysis always predicts some enhancement of the cracking strain of a transverse-ply in a laminate over that of the failing strain of the ply when tested independently. Observed cracking strain values agree closely with those derived from the analysis. Other possible failure modes are described by the analysis and these depend on the laminate characteristics and the magnitude of the applied load. Various means of suppressing cracking in the transverse-plies are suggested.

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